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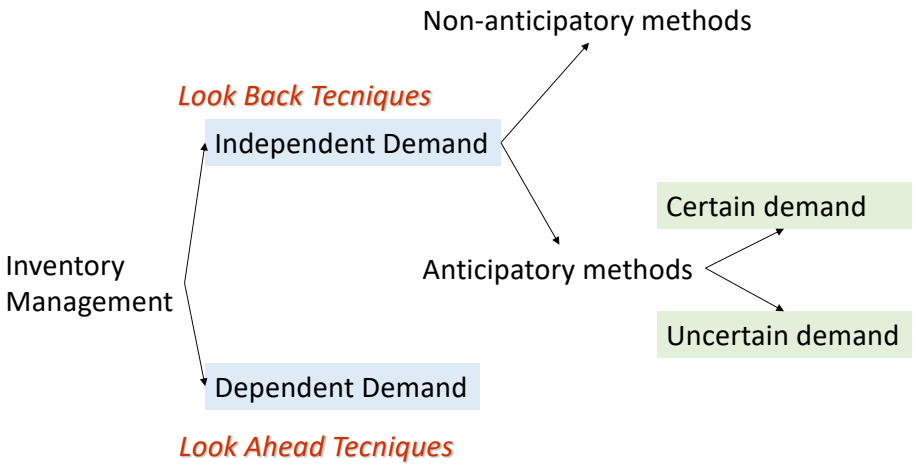
Inventory Management

Replenishment Models

v1. Spring 2021

Inventory Control Systems



Independent vs Dependent Demand

Independent demand

The demands for these items are unrelated to each other, or to activities that can be predicted with certainty

Dependent demand

The need for an item is a direct result of the need for some other item, usually an item of which it is a part. Also, when the demand for the item can be predicted with accuracy due to a schedule or specific activity

Inventory Management Models for Independent Demand

- **Quantity–based or "*Reorder Level*"**

  Placing of the order is conditional on the occurrence of the event:

  Inventory Position (IP) < Reorder Level (R)

  ***IP* = Inventory on Hand (*IOH*) + Inventory On Order (*OO*) - Back Orders (*BO*)**

  - *OO*: ordered quantities – *in transit* (Pipeline Inventory)
  - *BO*: quantity in stock but already allocated due to previous pending orders

- **Time-based or "*Reorder Cycle*"**

  Placing of the order is conditional on reaching a predetermined point in time

Inventory Replenishment Models

Goal

- Find an optimal policy for managing inventory

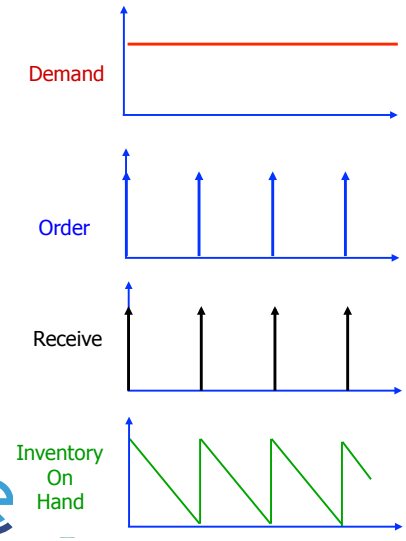
Policy consists of:

- **How much to order (Q)**
- **When to order (T)**
  1. Quantity-based models --> when inventory is a given level
  2. Time-based models --> every T time units
  3. Mixed

Sizing problem: how much and when to buy!

1. Economic Order Quantity (EOQ)

(look back technique)



Model Assumptions (EOQ)

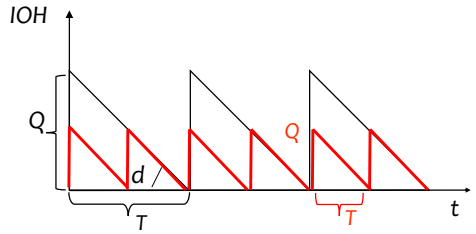
- Demand is uniform and deterministic
- Lead time is instantaneous (L=0)
- Total amount ordered is received

Economic Order Quantity (*Harris, 1913*)

Assumptions

- Demand for the product is constant and uniform throughout the period D [u/year] = const
- Price per unit of product is constant C_a [€/u] = const
- Inventory holding cost is constant C_m [€/(u·period)] = const
- Inventory holding cost is based on average inventory $A_Inv = Q/2$
- Shortage cost is constant C_s [€/u] = const
- Lead time (time from ordering to receipt) is constant L = const = 0
- Inventory has infinite capacity
- Replenishment time of the stock is negligible
- Ordering or setup costs are constant C_L [€/order] = const
- Ordering (or setup cost) is proportional to the actual number of orders placed n_L $\quad n_L = \frac{D}{Q}$

Economic Order Quantity (EOQ)



d = cost
L = 0

$$C_{tot} = \cancel{C_{tot,acquisto}} + C_{tot,emiss.ordini} + C_{tot,mantenimento} + \cancel{C_{tot,stockout}} \quad [€/anno]$$

$$C_{tot} = C_a D + \frac{D}{Q} C_L + \frac{Q}{2} C_m + C_s E[u\ short] \quad \Rightarrow \quad \mathrm{TRC} = \frac{D}{Q} C_L + \frac{Q}{2} C_m$$

$$\frac{\partial C_{tot}}{\partial Q} = \frac{-C_L D}{Q^2} + \frac{C_m}{2} = 0$$

$$\frac{\partial^2 C_{tot}}{\partial^2 Q} = \frac{2 C_L D}{Q^3} > 0$$

$$Q^* = EOQ = \sqrt{\frac{2 C_L D}{C_m}}$$

Second order condition is always true!
Q is a global minimum*

Economic Order Quantity (EOQ)

- Inventory Policy:
  - Order Q* units when *IOH = 0*
  - Order Q* units every T* time periods

$$T^* = \frac{Q^*}{D} = \sqrt{\frac{2C_L}{C_m D}}$$

$$TRC(Q^*) = \sqrt{2C_L C_m D}$$

$$TC(Q^*) = C_a D + \sqrt{2C_L C_m D}$$

Example

- Demand = 2000 u/year
- Cost of placing an order = 500 €/order
- Cost of product = 50 €/u
- Holding cost = 25% of unit cost per year
- Selling price of product = 75 €/u

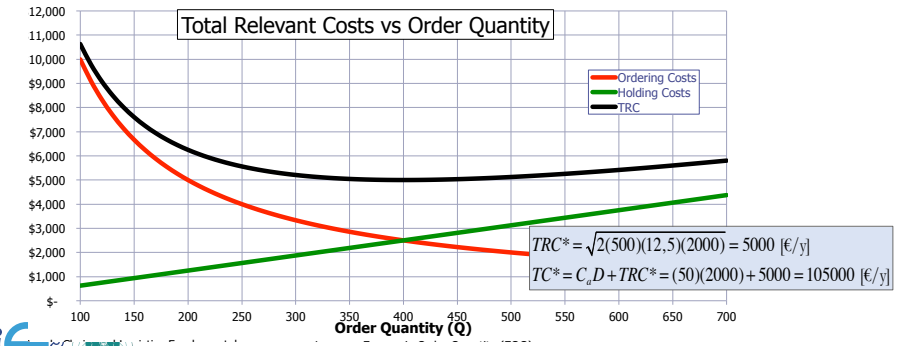
Policy

$$Q^* = EOQ = \sqrt{\frac{2C_L D}{C_m}} = \sqrt{\frac{2(500)(2000)}{(0{,}25)(50)}} = 400 \text{ [u/ord]}$$

$$T^* = \frac{Q^*}{D} = \frac{400}{2000} = 0{,}2 \text{ [y/ord]} = 73 \text{ [d/ord]}$$

That is, when *IOH*=0!!

$$n_L = \frac{D}{Q^*} = \frac{1}{T^*} = \frac{2000}{400} = 5 \text{ [ord/y]}$$



$$TRC^* = \sqrt{2(500)(12{,}5)(2000)} = 5000 \text{ [€/y]}$$

$$TC^* = C_a D + TRC^* = (50)(2000) + 5000 = 105000 \text{ [€/y]}$$

EOQ Sensitivity – *a robust solution*

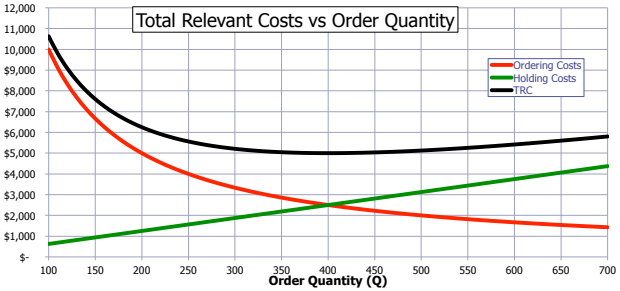
- What happens if we don't use the optimum quantity?
- What is preferable: Q>Q* o Q<Q*?

Q	Ordering Costs ($c_L D/Q$)	Holding Costs ($c_e Q/2$)	Total Relevant Costs	Q/Q*	TRC/TRC*
800	$1,250	$5,000	$6,250	200%	125%
600	$1,667	$3,750	$5,417	150%	108.3%
Q*=400	$2,500	$2,500	$5,000	--	--
200	$5,000	$1,250	$6,250	50%	125%
20	$50,000	$125	$50,125	5%	1002.5%

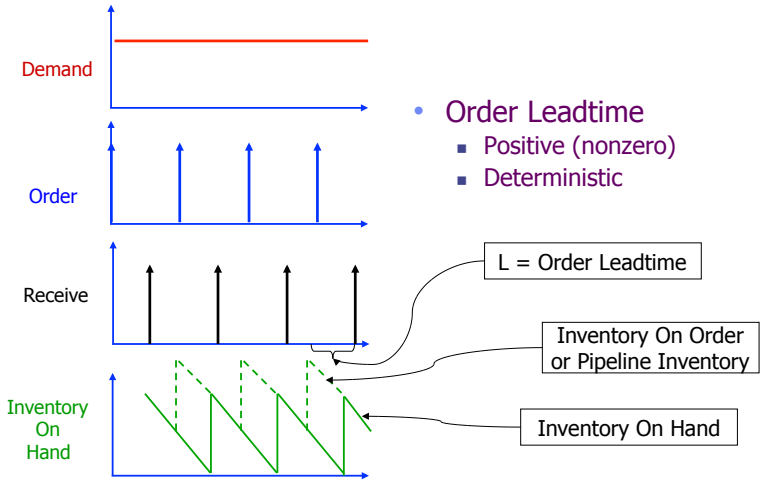
- … and what if we made errors in demand forecasts?

is relatively insensitive to changes in . . .

- rounding of order quantities (Q)
- rounding of order cycle time (T)
- errors in forecast (D)
- errors in cost parameters



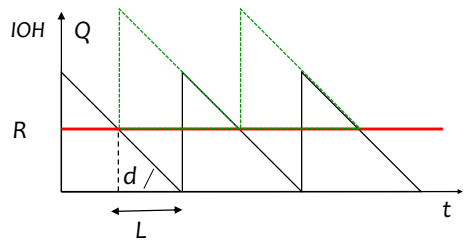
EOQ model extension: Lead Time *L*>0



- Order Leadtime
  - Positive (nonzero)
  - Deterministic

What is the average inventory in the system?
Cycle Stock + Pipeline = Q/2 + d L
Need to consider who owns it and when (on-hand vs. system)

EOQ model extension: Lead Time *L*>0



d = cost

L = cost

R = d · L

must be in congruent units!

$$C_{tot} = C_{tot,acquisto} + C_{tot,emiss.ordini} + C_{tot,mantenimento} + C_{tot,stockout} \quad [€/anno]$$

$$C_{tot} = C_a D + \frac{D}{Q} C_L + \left(\frac{Q}{2} + d\,L\right) C_m + C_s E[u\ short]$$

Cycle Stock Pipeline Inventory!

$$\frac{\partial C_{tot}}{\partial Q} = 0 \Rightarrow \boxed{Q^* = EOQ = \sqrt{\frac{2 C_L D}{C_m}}}$$

What does it change?

Nothing!

Economic Order Quantity (EOQ) – *L*>0

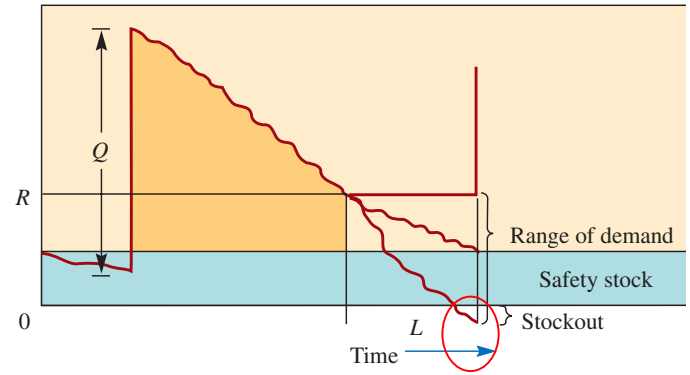
- Inventory Policy:
  - Order Q* units when *IOH = d·L*
  - Order Q* units every T* time periods

$$\boxed{T^* = \frac{Q^*}{D} = \sqrt{\frac{2C_L}{C_m D}}}$$

Demand over Leadtime
Represents the expected value of consumption during the order leadtime (*L*)

Extension – Demand Uncertainty

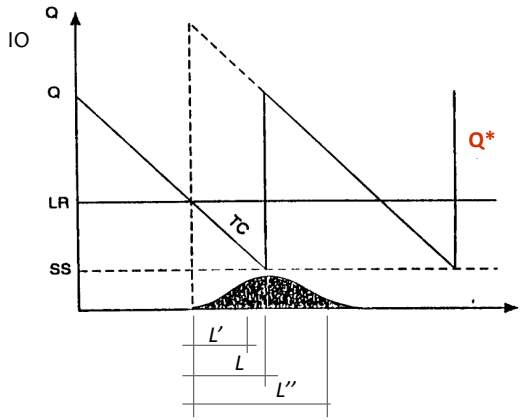
1. Overstock: excess of inventory (*d'<d*)
2. Stockout: lack of inventory (shortage) (*d''>d*)



Uncertainties in demand can be dealt with by using Safety Stocks (SS)

Extension – Lead Time Uncertainty

1. Overstock: excess of inventory (*L'<L*)
2. Stockout: lack of inventory (shortage) (*L''>L*)



Uncertainties in Lead Time can be dealt with by using Safety Stocks (SS)

Safety Stock (SS)

The Safety Stock depends on:

1. stability of demand or internal consumption
2. reliability of the supply system
3. the desired level of service (LS) defined as the maximum percentage of out-of-stock periods admitted

The **variability** of the demand (*d*) and lead time (*L*) must be known to establish the *SS* at a given Service Level.

We assume that the random variables d_i [demand of the *i-th* period] and *L* [number of periods] are characterised by a NORMAL frequency distribution (variability is due to purely random phenomena) with expected value $\bar{d}$, $\bar{L}$ and standard deviation σ_{di} and σ_L respectively

Safety Stock calculation

$SS = k \cdot \sigma_{dL}$

- σ_{dL} standard deviation of consumption during lead time
- *k* the number of standard deviations of safety stock associated to a specific Service Level (probability of not stocking out during the lead time) → *k* = inv.norm.st(*LS*) or table

k is the value of the **standardised variable** (characterised by mean = 0 and variance = 1) corresponding to a cumulative probability value equal to the desired *Service Level (LS)*

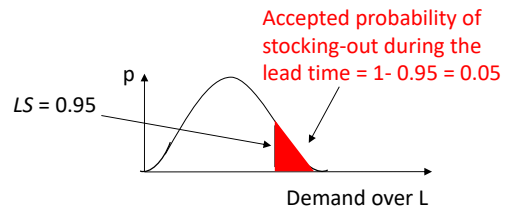
LS = 50% → *k = 0*

LS = 90% → *k = 1.28*

LS = 95% → *k = 1.65*

LS = 98% → *k = 2.06*

LS = 99,9% → *k = 3.09*



Safety Stock calculation

1. Variable demand – Deterministic Lead time

Variability in demand (and/or supply time) occurs over time and therefore the law of compound probability applies!
The standard deviation of a series of independent occurrences is equal to the square root of the sum of the variances

$$\sigma_{dL} = \sqrt{\sum_{i=1}^{L} \sigma_i^2}$$

Hp: $\sigma_i = \sigma_d = const\ \forall i \Longrightarrow \sigma_{dL} = \sqrt{\sigma_d^2 L}$

$$SS = k\sigma_{dL} = k\ \sigma_d \sqrt{L}$$

Safety Stock calculation

2. Deterministic demand – Variable Lead time

$$Pr\{L \le L_{max}\} = Pr\{z \le k\}$$
$$where\ z = \frac{L - \bar{L}}{\sigma_L}$$
$$SS = (L_{max} - \bar{L})\ \bar{d} = k\sigma_L \bar{d}$$

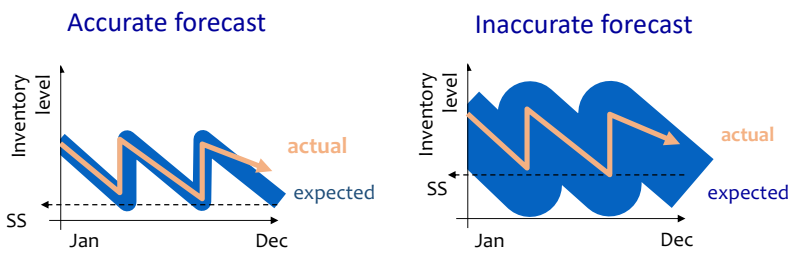
$$SS = k\sigma_L \bar{d}$$

3. Variable demand – Variable Lead time *(Hadley and Within Model, 1963)*

Hp: $\bar{d}_i = \bar{d},\ \forall i = 1, \dots, L$

$$SS = k\sqrt{\sigma_L^2 \bar{d}^2 + \sigma_d^2\ \bar{L}}$$

Safety Stock calculation



$$SS = k\sqrt{\sigma_L^2 \bar{d}^2 + \sigma_d^2 \bar{L}}$$

Continuous Review Policy (s, Q)

- This is also known as the **Order-Point, Order-Quantity policy**
- The policy is:
- "Order Q* units when Inventory Position is less than the re-order point s"

  - Reorder Point: $s = E(d_L) + SS = \bar{d}\bar{L} + k\sigma_{dL}$
  - Order Quantity: Q^*