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Inventory Management

Replenishment Models

v1. Spring 2021

Inventory Replenishment Models

Goal

- Find an optimal policy for managing inventory

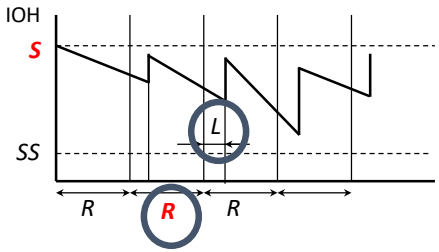
Policy consists of:

- **How much to order (Q)**
- **When to order (T)**
  1. Quantity-based models --> when inventory is a given level
  2. Time-based models --> every T time units
  3. Mixed

Sizing problem: how much and when to buy!

2. Periodic Order Quantity (POQ)

Reordering is at regular time intervals!



The safety stock is on average higher!

- S target level of inventory
- d demand per period with mean $\bar{d}$ and standard deviation σ_d
- R revision interval between two consecutive orders
- L supply time (assumed constant)
- q order quantity (***variable at each revision cycle***)

$$S = \bar{d}(R + L) + SS_{R+L}$$

2. Periodic Review Policy (R, S)

Since the period demand is stochastically variable, while the review period and the lead time constant, results:

$$\sigma_{R+L} = \sigma_d\sqrt{(R + L)}$$

Standard deviation of the demand over the time interval (R+L)

Safety Stock

$$SS = k\sigma_{R+L} = k\sigma_d\sqrt{(R + L)}$$

Order quantity

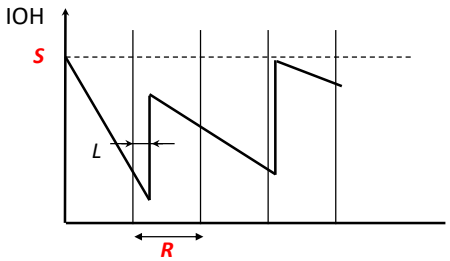
$$q = S - IP = \bar{d}(R + L) + k\sigma_{R+L} - IP$$

Periodic Review Policy (R, S)
Order up to S units every R time periods

Mean Inventory

$$A_INV = SS + \frac{\bar{d}R}{2}$$

2b. (R, S) policy with minimum *Q*



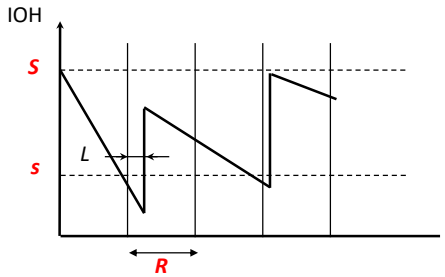
The system forces stock control at fixed intervals (R) and requires a replenishment order if the level falls below a specific value.

S maximum estimated inventory by optimising storage and order costs

Q_{min} minimum order quantity

Order quantity (*q*) = (*S* - *IP*) is issued if this difference is greater than Q_{min} otherwise, the order is not released, moving to the next interval *R*

2c. Order-up-to policy (Min/Max)(R, s, S)



- the variable order quantity is issued at fixed reorder intervals (*R*) if availability is below a minimum inventory - reorder level (*s*)
- the order quantity is equal to the difference between a maximum level (*S*) and the inventory position at the time the order is released

Policy: How much to order and when

- **EOQ Policy** – event based, deterministic demand, single period
  - Order Q* every T* time periods
  - Order Q* when *IP = d L*
- **Continuous Review Policy (s, Q)** – event based, stochastic demand and lead time
  - Order Q* when $IP \leq s$, $s = d\,L + k\sigma_{dL}$
- **Periodic Review Policy (R, S)** – time based, infinite planning horizon
  - Order up to S units every R time periods

Recall:
Inventory Position (*IP*) = Inventory on Hand (*IOH*) + Inventory on Order (*OO*) – Backorders (*BO*)
Demand over Leadtime = *d L*

How do we choose the 'right' policy?

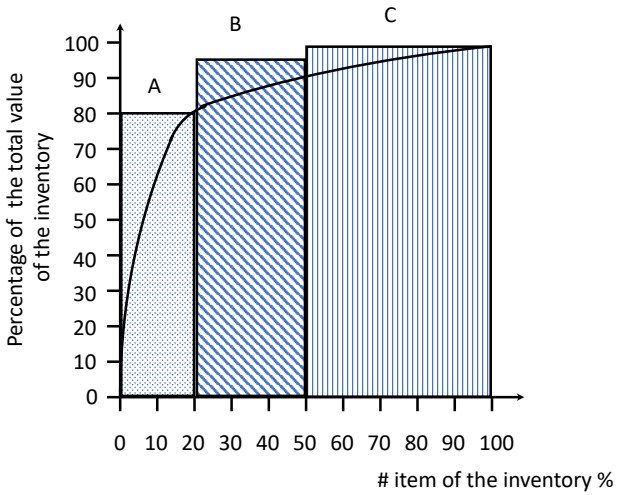
Pareto Principle - ABC Classification

"In any given set of factors to be controlled, **a small fraction, in numbers, has a significant influence in terms of effect**.

In contrast, the vast majority of factors, again in numbers, have relatively less significance in terms of effect."

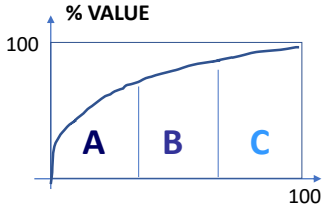
(Parigi 1848 - Celigny 1923)

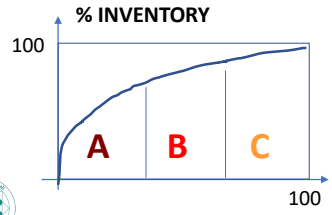
Pareto Principle - ABC Classification



ABC Classification – Matrix (Q-V)

the analysis of the 9 subclasses of the matrix gives several useful indications to choose the inventory management policy





		VALUE		
		A	B	C
INVENTORY	A	1	2	3
	B	4	5	6
	C	7	8	9

ABC class and Inventory Policy

Tends to be Quantity-based

		VALUE		
		A	B	C
INVENTORY	A	1	2	3
	B	4	5	6
	C	7	8	9

Tends to be Time-based

Actively manage - ACTIVE
Continuous Review Policy (s ,Q)
Periodic Review Policy (R, s, S)

Manage by exception - AUTOMATIC
EOQ e (s, Q) Policy

Set & forget - PASSIVE
Periodic Review Policy (R,S)

How to define the Inventory Policy

- The EOQ model is only valid for independent item reorders: joint item reorders are not allowed.
- The EOQ model allows limited seasonal variations and requires continuous monitoring.
- The EOQ model is more time-consuming, because each withdrawal and load is accounted for
- The (*s, Q*) model is suitable for more expensive items (class A of an ABC analysis), because the average amount of stock is lower and because there is tighter control and therefore a faster reaction to potential stock-outs
- Reorder interval models (*R*) require higher safety stock because it must also protect against the danger of out-of-stocking during *R*

Summary

- Analyze how different inventory control systems work
- Establishing Safety Stock Levels
- ABC Classification using the *Pareto Principle*

- References:
  Ch. 20 Jacobs, Chase